Chapter 1

1) \[ \lim_{x \to \frac{1}{2}} \frac{2x^2 + 5x - 3}{-7x + 2 + 6x^2} = \lim_{x \to \frac{1}{2}} \frac{(2x-1)(x+3)}{(3x-2)(2x-1)} = \lim_{x \to \frac{1}{2}} \frac{x+3}{2x-2} = \frac{3.5}{-1} \]

2) \[ \lim_{x \to 2} \frac{x^2 - x - 2}{(x-2)^2} = \lim_{x \to 2} \frac{(x-2)(x+1)}{(x-2)^2} = \lim_{x \to 2} \frac{x+1}{x-2} = 0 \quad \text{since} \quad \lim_{x \to 2} (x^2 - x - 2) = \infty \quad \text{and} \quad \lim_{x \to 2} (x-2)^2 = 0 \]

3) \[ \lim_{x \to 0} \frac{\sin x}{2x} = \lim_{x \to 0} \frac{1}{2} \cdot \frac{\sin x}{x} = \frac{1}{2} \cdot 1 = \frac{1}{2} \]

1) \[ \lim_{x \to 0} x^2 \sin \left( \frac{1}{x^2} \right) = 0. \text{ Show why this is true.} \]

Since \(-1 \leq \sin x \leq 1\), then \(-1 \leq \sin \frac{1}{x^2} \leq 1\) also.

Multiply through by \(x^2\), which is positive as \(x \to 0\) and we get

\[-x^2 \leq x^2 \sin \frac{1}{x^2} \leq x^2.\]

Then since \(\lim_{x \to 0} (-x^2) = 0 = \lim_{x \to 0} x^2\), by sandwich theorem,

\(\lim_{x \to 0} x^2 \sin \frac{1}{x^2} = 0\).

5. Find the vertical and horizontal asymptotes for the following, state any discontinuities and the type of discontinuity.

\[ f(x) = \frac{(x-2)(x+3)}{(2x+3)(x+3)} \]

Vertical Asymptote (VA): \(x = -\frac{3}{2}\) only since at \(x = -3\) there is a hole, not a vertical asymptote.

\[ \lim_{x \to -\frac{3}{2}} f(x) = \frac{1}{2} \quad \text{and} \quad \lim_{x \to -\frac{3}{2}} f(x) = \frac{1}{2} \quad \text{so} \quad \text{HA:} \quad y = \frac{1}{2} \]

Find limit: \(f(x) = \frac{5}{x-4}\) when \(x \to 4^{-}\)

\[ \lim_{x \to 4^{-}} \frac{5}{x-4} = -\infty \]
Given the graph at the right, discuss its continuity and the functions limit at \( x = 0, 1, 2, 3 \) or as \( x \) approaches 0, 1, 2, 3.

Continues on \([0, 2) \cup (2, 3] \cup (3, \infty)\).

Point discontinuity at \( x = 2 \).

Jump discontinuity at \( x = 3 \).

Cont. at \( x = 0 \) only from the right.

\( \lim_{x \to 0^+} f(x) = 0 \), but \( \lim_{x \to 0^-} f(x) = \text{DNE} \)

\( \lim_{x \to 1^-} f(x) = 1.5 \)

\( \lim_{x \to 1^+} f(x) = 2 = \lim_{x \to 2^-} f(x) \), so \( \lim_{x \to 2^-} f(x) = 2 \)

\( \lim_{x \to 3^+} f(x) = 1 \) \( \Rightarrow \) \( \lim_{x \to 3^-} f(x) = \text{DNE} \)

\( \lim_{x \to 3} f(x) = 2 \) \( \Rightarrow \) \( \lim_{x \to 3} f(x) = \text{DNE} \)