The slope of the normal line at \((x_1, y_1)\) is \(-\frac{a^2 y_1}{b^2 x_1}\).

Since the normal line passes through \((0, 0)\) and \((x_1, y_1)\), its slope must be \(-\frac{y_1}{x_1}\).

Now, \(\frac{a^2 y_1}{b^2 x_1} = \frac{y_1}{x_1} \Rightarrow a^2 = b^2\). Thus, the ellipse is a circle.

52. Without loss of generality, let the hyperbola have the equation \(\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1\), the coordinates of \(P\) be \((x_1, y_1)\), and the equations of the asymptotes be \(y = \pm \frac{b}{a}x\).

From Exercise 50, the equation of the tangent line is \(\frac{x_1 x}{a^2} - \frac{y_1 y}{b^2} = 1\). First, consider the point of intersection with the asymptote \(y = -\frac{b}{a}x\) at \(Q(x_Q, y_Q)\). Solving the equations simultaneously gives \(\frac{x_1 x}{a^2} - \left(-\frac{b}{a}x\right)\left(\frac{y_1}{b}\right) = 1 \Rightarrow x_Q = \frac{a^2 b}{bx_1 + a y_1}\). Then, \(y_Q = -\frac{b}{a}x_Q = -\frac{a b^2}{bx_1 + a y_1}\). Second, consider the point of intersection with the asymptote \(y = \frac{b}{a}x\) at \(R(x_R, y_R)\). In a similar manner, \(x_R = \frac{a^2 b}{bx_1 - a y_1}\) and \(y_R = \frac{a b^2}{bx_1 - a y_1}\). The first coordinate for the midpoint of \(QR\) is \(\frac{1}{2}(x_Q + x_R) = \frac{1}{2}\left[\frac{a^2 b^2}{bx_1} - \frac{a^2 b^2}{a y_1}\right] = \frac{a^2 b^2 x_1}{a^2 b^2 - b^2 y_1}\) (since \((x_1, y_1)\) are on the hyperbola) \(= x_1\).

In a similar manner, \(\frac{1}{2}(y_Q + y_R) = y_1\). Hence, the midpoint is \(P(x_1, y_1)\).
10. Let \( z \) denote the diameter.

\[
A = \frac{1}{4} \pi z^2 \Rightarrow \frac{dA}{dt} = \frac{1}{2} \pi z \frac{dz}{dt} = \frac{1}{2} \pi (30)(0.01) = 0.15 \pi \approx 0.471 \text{ cm}^2/\text{min}.
\]

10. \( A = \pi r^2 \Rightarrow \frac{dA}{dt} = 2\pi r \frac{dr}{dt} = 2\pi(150)(6) = 1800\pi \approx 5655 \text{ ft}^2/\text{min}.\)

11. \{ diameter \( = 18 \text{ in.} \Leftrightarrow \) radius \( = 9 \text{ in.} = \frac{3}{4} \text{ ft.}\}

\[
V = \frac{4}{3} \pi r^3 \Rightarrow \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{1}{4\pi r^2} \frac{dV}{dt} = \frac{1}{4\pi \left(\frac{3}{4}\right)^2} \frac{20}{9\pi} \approx 0.707 \text{ ft/min}.
\]

12. \( V = \frac{4}{3} \pi r^3 \Rightarrow \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} = 4\pi(10)^2\left(-\frac{4}{15}\right) = \frac{320\pi}{9} \approx -111.7 \text{ in.}^3/\text{min}.\)

13. Let \( z \) denote the distance between the base of the building and the bottom of the ladder, and \( y \) denote the distance between the base of the building and the top of the ladder. \( z^2 + y^2 = 400 \Rightarrow z = \sqrt{336} \text{ when } y = 8 \) and

\[
2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0 \Rightarrow \frac{dx}{dt} = -\frac{y}{x} \frac{dy}{dt} = -\frac{\sqrt{336}}{8}(3) = -\frac{3}{8}\sqrt{336} \approx -6.9 \text{ ft/sec}.
\]

14. Let \( z \) denote the distance of the first girl east of \( A \), \( y \) the distance of the second girl north of \( A \), and \( x \) the distance between the girls.

\[
z^2 = x^2 + y^2 \Rightarrow 2z \frac{dz}{dt} = 2x \frac{dx}{dt} + 2y \frac{dy}{dt} \Rightarrow \frac{dz}{dt} = \frac{1}{2} \left( \frac{dx}{dt} + \frac{dy}{dt} \right).
\]

\[
x = 10 \text{ ft/sec (120 sec)} = 1200 \text{ ft}, \ y = 8 \text{ ft/sec (60 sec)} = 480 \text{ ft} \Rightarrow z = 120\sqrt{116} \text{ ft}.
\]

\[
\frac{dx}{dt} = 10 \text{ and } \frac{dy}{dt} = 8 \Rightarrow \frac{dz}{dt} = \frac{1}{2} \frac{1}{120\sqrt{116}} \left[ 1200(10) + 480(8) \right] = \frac{132}{\sqrt{116}} \approx 12.3 \text{ ft/sec}.
\]

15. Let \( z \) denote the distance of the tip of the shadow from the base of the pole, \( y \) the distance of the boy from the base, and \( x \) the length of the shadow.

\[
z = x - y = \frac{dx}{dt} = \frac{dx}{dt} - \frac{dy}{dt}. \text{ By similar triangles, } \frac{x}{16} = \frac{x - y}{5} = \frac{z}{16} y \Rightarrow
\]

\[
\frac{dx}{dt} = \frac{16}{11} \frac{dy}{dt} = \frac{16}{11}(4) \approx 5.82 \text{ ft/sec}. \text{ Thus, } \frac{dx}{dt} = \frac{64}{11} - 4 = \frac{20}{11} \approx 1.82 \text{ ft/sec}.
\]

16. Let \( z \) denote the horizontal distance between the bow of the boat and the dock and \( L \) the length of rope between the boat and the pulley.

\[
L^2 = z^2 + y^2 \Rightarrow 2L \frac{dL}{dt} = 2z \frac{dx}{dt} \Rightarrow \frac{dx}{dt} = \frac{L}{z} \frac{dL}{dt} = \frac{\sqrt{674}}{25}(-2) = -\frac{2}{25}\sqrt{674} \approx -2.08 \text{ ft/sec (negative since } \frac{z}{x} \text{ is decreasing)}.
\]

17. Let \( T \) denote the thickness of the ice and note that the radius is 120 in. The volume of the ice (outer hemisphere - inner hemisphere) is

\[
V = \frac{2}{3}\pi(120 + T)^3 - \frac{2}{3}\pi(120)^3 \Rightarrow \frac{dV}{dt} = 2\pi(120 + T)^2 \frac{dT}{dt} = 2\pi(120 + 2)^2\left(-\frac{1}{4}\right) = -7442\pi \approx -23,380 \text{ in.}^3/\text{hr}.
\]

18. Since \( r = h \), \( V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi h^3 \).

\[
\frac{dV}{dt} = \pi h^2 \frac{dh}{dt} = \pi(10)^2(6) = 600\pi \approx 1885 \text{ in.}^3/\text{min}.
\]
19. Let \( L \) denote the length of string and \( x \) the horizontal distance of the kite from the boy. \( L^2 = x^2 + 100^2 \implies 2L \frac{dL}{dt} = 2x \frac{dx}{dt} \Rightarrow \frac{dx}{dt} = \frac{L \frac{dL}{dt}}{x} = \frac{125}{15} = \frac{10}{3} \approx 3.33 \text{ ft/sec.} \)

20. Let \( h \) denote the height of the balloon and \( L \) the length of the rope. \( L^2 = h^2 + 20^2 \implies 2L \frac{dL}{dt} = 2h \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{L \frac{dL}{dt}}{h} = \frac{\frac{500}{\sqrt{249,600}}}{5} = \frac{2500}{\sqrt{249,600}} \approx 5.00 \text{ ft/sec.} \)

21. \( pv = c \implies p \frac{dv}{dt} + v \frac{dp}{dt} = 0 \Rightarrow \frac{dp}{dt} = -\frac{v}{p} \frac{dv}{dt} = -\frac{75}{50}(-2) = 5 \text{ in}^3/\text{min (increasing)} \).

22. Let \( z \) denote the diameter of the cable in inches.

Hence, \( A = \pi z (1200) \) is the curved surface area.

\[
\frac{dA}{dt} = 1200 \pi \frac{dz}{dt} \Rightarrow \frac{dz}{dt} = \frac{1}{1200 \pi} \frac{dA}{dt} = \frac{1}{1200 \pi} (750) = \frac{5}{8 \pi} \approx 0.1989 \text{ in./yr.}
\]

23. Let \( h \) denote the depth of the water. The area of the submerged triangular portion is

\[
A = \frac{1}{2} \left( \frac{2h}{\sqrt{3}} \right) h = \frac{h^2}{\sqrt{3}} \implies \frac{dV}{dt} = 8A = \frac{8h^2}{\sqrt{3}} \Rightarrow \frac{dh}{dt} = \frac{1}{16h} \frac{dV}{dt} = \frac{1}{16h} \left( \frac{5}{92} \right) = \frac{15}{32} \approx 0.81 \text{ ft/min.}
\]

24. Using the same notation as in Exercise 23, \( A = \frac{1}{2} h^2 \) and \( V = 4h^3 \Rightarrow \frac{dV}{dt} = 8Ah \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{1}{8h} \frac{dV}{dt} = \frac{1}{8} \left( \frac{5}{92} \right) = 0.0375 \text{ ft/min.} \)

25. Let \( x \) denote the length of a side.

\[
A = \frac{1}{4} x^2 \Rightarrow \frac{dA}{dt} = \frac{1}{2} x \frac{dx}{dt} \Rightarrow \frac{dx}{dt} = \frac{2}{\sqrt{3} x} \frac{dA}{dt}. \quad A = 200, \quad x = \left( \frac{800}{3} \right)^{1/2}, \quad \text{and} \quad \frac{dA}{dt} = -4 \Rightarrow \frac{dx}{dt} = \frac{2}{\sqrt{3} \left( \frac{800}{3} \right)^{1/2}} (-4) = -\sqrt{2} \approx -0.2149 \text{ cm/min.}
\]

26. \( V = \frac{4}{3} \pi r^3 \Rightarrow \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{1}{4\pi r^2} \frac{dV}{dt}. \quad V = 400, \quad r = \sqrt[3]{300} \pi, \quad \text{and} \quad \frac{dV}{dt} = -10 \Rightarrow \frac{dr}{dt} = \frac{1}{4\pi \left( \sqrt[3]{300} \pi \right)^2} (-10) = -\frac{5}{2} \frac{\pi}{300} \approx -0.038 \text{ ft/min.} \)

27. \( C = 2\pi r \Rightarrow \frac{dC}{dt} = 2\pi \frac{dr}{dt} = 2\pi (0.5) = \pi \approx 3.14 \text{ m/sec.} \)

Note: \( C \) is linear in \( r \) so its rate of change is constant.

28. Let \( x \) denote the runner's distance from third base and \( h \) her distance from home plate. \( h^2 = x^2 + 60^2 \Rightarrow 2h \frac{dh}{dt} = 2x \frac{dx}{dt} \Rightarrow \frac{dh}{dt} = \frac{x}{h} \frac{dx}{dt} = \frac{20}{\sqrt{4000}} (-24) = \frac{-24}{10} \approx -7.59 \text{ ft/sec.} \)

29. \( \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow \frac{1}{R} \frac{dR}{dt} = -\frac{1}{R_1^2} \frac{dR_1}{dt} - \frac{1}{R_2^2} \frac{dR_2}{dt}. \quad R_1 = 30 \text{ and } R_2 = 90 \Rightarrow R = \frac{45}{2} \text{ ft.} \quad \frac{dR_1}{dt} = 0.01 \text{ and } \frac{dR_2}{dt} = 0.02 \Rightarrow \frac{dR}{dt} = -\left( \frac{45}{2} \right)^2 \left[ -\frac{1}{(30)^2} (100) - \frac{1}{(90)^2} (0) \right] = \frac{11}{1600} = 0.0006875 \text{ ohm/sec.} \)